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Hands-on Tutorial on Optimization

Exercise Sheet: Cheese Delivery

Since you proved very helpful and loyal to Baron von Due during the last week, he has one final task for you. Plus, his Dutch assistant Mr. Gouda left von Due under suspicious circumstances involving the daughter of M.N. Taler and the dream of his own cheese empire in the Netherlands. Therefore von Due needs you for perfecting his cheese empire.

Von Due's famous cheese mixture is now part of Swiss lifestyle and enjoyed in every major city. Not only by citizens, but also by a increasing number of tourists. Only a minor part of this success is due to the new name: Fondue. Of course, now that everybody connects this perfect dish with Baron von Due, he sees a huge business opportunity: Instead of having another company earn money selling his cheese to the people, he plans to found restaurants all over the cities. There, people can meet, enjoy the best wine selected by his dear friend Monsieur Baguette and – accidentally of course – spend a fortune on having the chance to enjoy the original Fondue in a very exclusive atmosphere and maybe meet Baron von Due in person.

For this plan to work, von Due wants you to figure out the cheapest way of transporting the melted cheese mixture from the end point of the KäMaPi to the various locations where von Due plans to set up his restaurants. For the transport within each city, von Due plans to use special trucks (KäMatrucks) that look a lot like concrete mixing trucks, but can also keep the perfect temperature for the cheese mixture. The plan should be set such that it the same plan can then be identically executed for each day. Therefore, every vehicle should always start at the endpoint of the KäMaPi and return to the same point after its tour.

Von Due asks you to develop a solution that satisfies the demand of each restaurant, does not exceed the capacity of any truck, and minimizes the total cost. The total cost consists of the distribution cost and some possible penalty cost if the cheese arrives to late at the restaurants. Since you are familiar with combinatorial optimization, you immediately recognize the problem as a variant of the Vehicle Routing Problem.

Before leaving, Mr. Gouda could be convinced to collect all the data for the various cities and to set up the data files you need for your assignment. For each city, he created a separate data file and you should solve the problems separately for each city. Within one city, the set `locations` contains the restaurants as well as the endpoint of the respective KäMaPi denoted by `0`. Besides the x and y coordinates of each location (`xcoor[j]` and `ycoor[j]`), these data files also contain the travel times `t[i,j]` in minutes and distances `d[i,j]` in kilometer between any two locations i and j. Of course, Mr. Gouda also estimated the demand `dem[j]` of each restaurant and - depending on the city - he estimated the cost for delivering the cheese *after* the respective restaurant opened. Delivering before opening does not incur any extra cost. In order to enhance customer experience, every restaurant opens at 7pm, while the point in time when the trucks can leave the KäMaPi depends on the city and is given in the data file as `start`.

The truck specifications also depend on the city, so Mr. Gouda included them in the data files as well. The set of vehicles is given by `trucks`. Using a truck costs a fixed amount of `fix` euros and additionally `var_min` euros per minute as well as `var_km` euros per km. The capacity of a truck is denoted by `cap`.

For each of the following exercises formulate the solution as an (I)LP and solve the problem using CPLEX. Note that the difficulty of the problems increases with the number of the exercise as well as with the initial of the city. Exercises 1 to 4 should only be solved for Bern, Genf, Lausanna, and Zuerich while exercise 5 is tailored for St. Moritz.

For handing in your solutions, please take the following remarks into account.

Assignment:

- Not everyone may be able to solve all the assignments for each of the cities. We do expect you to solve each assignment for some of them and, of course, for as many as possible.
- HINT: This Cheese Delivery Problem is a variant of the well-known Vehicle Routing Problem (VRP).

Report:

- To make your write up better readable, each variable / parameter should be exactly one letter and possibly some indices. Please explain which letter denotes which variable / parameter.
- Explain each model including its constraints and argue why this solves the corresponding exercise. That is, explain what each of the constraints should do and how this is achieved.
- The solutions you found with CPLEX should be interpreted, i.e., what does each route look like, what is the penalty incurred by the vehicles, etc. In particular, we do not accept the solution files generated by CPLEX. The coordinates of each location may be used to plot the solutions in the report.
- If you where not able to solve some assignment for some cities, mention which, what problems you ran into, and what you tried to resolve those problems.
- If you use any methods that go beyond linear programming, clearly state what you did and how you did this. This includes setting specific options for the solver and any ways that problems may be split up.
- If you use any sources beyond the course slides and exercises, you need to add proper citations in the report.

Code:

- Please choose the names of parameters as indicated by the data files and the variables in a manner consistent with your report.

Exercise 1 (Unit penalty)

For any restaurant that receives the cheese delivery after 7pm, Mr. Gouda estimated a cost of `unit_penalty` euros that should reflect the unhappiness of the customers. To simplify the problem in the first phase, von Due does not allow several trucks to serve the same restaurant. This means that the demand of any restaurant has to be satisfied by exactly one truck.

Exercise 2 (Penalty cost per minute)

After Baron von Due successfully rolled out his chain of restaurants, he quickly figures out that the unit penalty assumption of Mr. Gouda is to conservative as a restaurant that serves cheese starting at 7:15pm has happier customers than one that only starts serving cheese at 8:45pm. After a short market survey, you found the new parameter `minute_penalty` that describes the penalty cost incurred by a restaurant with late delivery. Indeed, a penalty per minute is more appropriate to the problem at hand, so from now on assume this type of penalty cost. Again, each restaurant should be served by one vehicle.

Exercise 3 (Partial demand)

The citizens of the *Chosen Cities* as they call the cities with Fondue restaurants are so happy with Baron von Due that they do not mind the sight of the KäMatrucks anymore. Thus, Baron von Due follows your suggestion to remove the constraint that exactly one truck has to serve a restaurant. From now on, any number of vehicles can serve one restaurant. However, as von Due does not completely trust his drivers, the amount of cheese delivered to a restaurant by a particular truck has to be integer in kilograms, i.e., a truck cannot deliver 125.6kg to one restaurant but either 125kg or 126kg.

Exercise 4 (Two types of trucks)

With your solutions, Baron von Due does not have to wait long for the success - and the money. Baron von Due always is a businessman ahead of his time and he wants to keep it this way. His newest idea to improve the revenue of his cheese empire is to use two different types of trucks. He now has twice as many trucks. The smaller one has less capacity but is also cheaper in operation.

Exercise 5 (St. Moritz)

Baron von Due feels at home among the extremely wealthy tourists in the famous ski resort St. Moritz. Therefore, he ordered three different KäMaPis to be built in order to increase customer satisfaction (and, thus, revenue). This means, there are now three different points set up where the trucks can access the KäMaPi within St. Moritz. Unfortunately, Baron von Due is not concerned with the satisfaction of his drivers. He only requires that the number of trucks leaving one access point in the afternoon should be the same number that returns in the evening. Hence, one particular truck does not need to return to its starting point but to any of the three access points.