Approximation Algorithms

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Greedy Algorithms

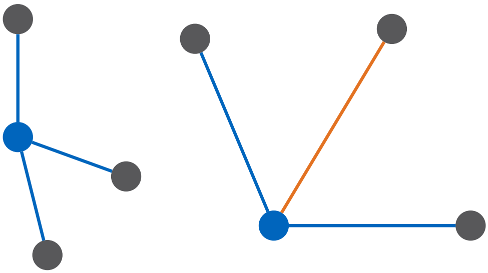
Example I: The metric k-Center Problem

The k-Center Problem

Input: metric space V with distances d_{ij} for $i, j \in V$ and some $k \in \mathbb{N}$ (metric: $d_{ii} = 0, d_{ij} = d_{ji} \geq 0$, and triangle ineq. $d_{ij} + d_{jk} \geq d_{ik}$)

Task: Find k centers in V, i.e., find $S \subseteq V$ with $|S| \leq k$.

Objective: minimize $\max_{i \in V} d(i, S)$ with $d(i, S) = \min_{s \in S} d(i, s)$



Greedy algorithm

Algorithm:

1. Let $S := \{v_0\}$ for some arbitrary $v_0 \in V$.
2. while $(|S| < k)$
   Let $v \in \text{argmax}\, d(v, S)$.
   Add v to S.
3. Return S.

Applications:

- clustering, finding (dis-)similarities in large data sets
- placing ATMs/warehouses in a city

Theorem

The greedy algorithm is a 2-approximation for k-Center.

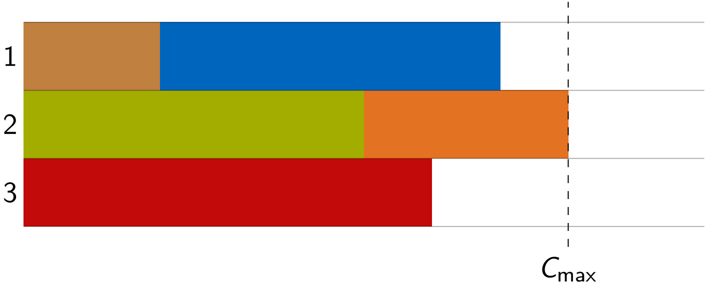
Greedy Algorithms

Example II: Scheduling on Identical Parallel Machines

Scheduling on Parallel Machines

Input: m identical machines,
n jobs with processing times $p_1, \ldots, p_n$

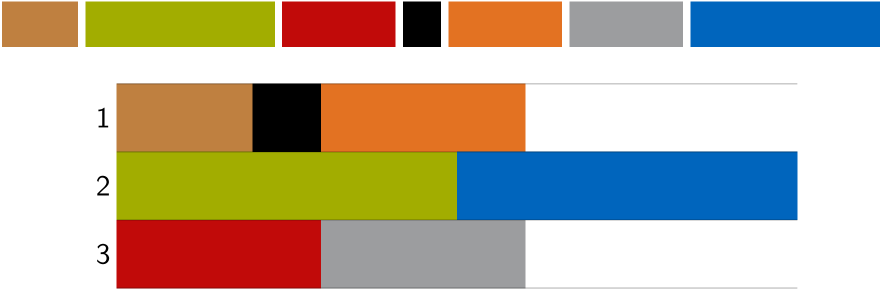
Task: assign each job $j \in [n]$ to a machine $i \in [m]$
minimizing the maximum load of any machine
$C_{\max} := \max_{i \in [m]} \sum_{j\,:\,j \to i} p_j$



List Scheduling

Algorithm:

1. For $j := 1$ to n
   Assign j to machine i with lowest load.



Theorem

List Scheduling is a 2-approximation for $P||C_{\max}$.

Longest Processing Time First

Algorithm:

1. Order jobs such that $p_1 \geq p_2 \geq \cdots \geq p_n$.
2. For $j := 1$ to n
   Assign j to machine i with lowest load.

Theorem

LPT List Scheduling is a 4/3-approximation for $P||C_{\max}$.