Universität Bremen | **FB3: Mathematik/Informatik**

Dr. Martin Böhm
Dr. Ruben Hoeksma
Prof. Dr. Nicole Megow

Summer 2019

Approximation Algorithms

Homework Sheet 7 (Deadline 09. 07. 2019 12:00 (paper) or 23:59 (email).)

All exercises must be done individually. Feel free (and encouraged) to discuss among each otherut each solution must be written independently. Two or more submissions found with exactly the same solution on any of the exercises will be awarded no points for the entire exercise sheet. The same holds in case of directly copying from any other source.

Exercise 1 (*8 points*). [Exercise 6.1 in WS] As with linear programs, semidefinite programs have duals. The dual for MAX CUT SDP is:

$$\min \frac{1}{2}\sum_{i<j} w_{ij} + \frac{1}{4}\sum_i \gamma_i$$
$$\text{s.t.} \qquad W + \operatorname{diag}(\gamma) \succeq 0$$

In this SDP, the matrix W is the symmetric matrix of edge weights and $\operatorname{diag}(\gamma)$ is the identity matrix except it has γ_i as the i-th entry on the diagonal (instead of 1).

Prove that the value of any feasible solution for this dual is an upper bound on the cost of any cut.

Exercise 2 (*12 points*). The same ideas from the 0.878-approximation for MAX CUT can be used for other problems where there are relations of exactly two variables. Consider the problem MAX E2 SAT, where we are given a list of clauses of size exactly two (such as $(x_i \vee \neg x_j)$ and we wish to find an assignment that maximizes the number of satisfied clauses.

a) Show that there is a quadratic integer program for MAX E2 SAT that uses only $\{-1,1\}$-variables. We recommend the following trick: alongside the "expected" variables, create a brand new variable $y_0 \in \{-1,1\}$ which will be declared to be the direction of `true` in your quadratic program. Again, it is useful to note that y_0 is a variable and it is not hardcoded to be 1.

   You should be able to find a quadratic $\{-1,1\}$-program with the objective function $\max \sum_{i,j} a_{i,j}(1 - y_i \cdot y_j) + b_{i,j}(1 + y_i \cdot y_j)$, where $a_{i,j}$ and $b_{i,j}$ are constants that arise from the instance (recall that for MAX CUT, $a_{i,j}$ were all $w_{ij} \cdot \frac{1}{2}$ and $b_{i,j}$ were all 0).

b) Show that there is a 0.878-approximation algorithm for any quadratic $\{-1,1\}$-program with the objective function $\max \sum_{i,j} a_{i,j}(1 - y_i \cdot y_j) + b_{i,j}(1 + y_i \cdot y_j)$ and no other constraints. You do not need to analyze any new functions other than those from the MAX CUT analysis.