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Approximation Algorithms

Exercise Sheet 6 (09.07.2019)

Exercise 1 Consider the minimum weighted vertex cover problem.

Definition 1 (Minimum weighted vertex cover)**.**

Input: *a graph $G = (V, E)$ and a weight function $w : V \to \mathbb{Q}_+$.*

Task: *find a subset $V' \subseteq V$ such that each edge in E has at least one endpoint in V'.*

Objective: *minimize $\sum_{v \in V'} w(v)$.*

The LP relaxation is as follows.

$$\begin{array}{lrll} \min & \sum_{v\in V} w_v \cdot x_v & & \\ \text{s.t.} & x_v + x_w & \geq 1, & \forall \{v,w\} \in E \\ & x_v & \geq 0, & \forall v \in V \end{array} \tag{VC}$$

Its dual linear program (a maximum weighted matching problem) is as follows. Recall that $\delta(v) := \{e \in E \,|\, e = (u,v), v \in V\}$.

$$\begin{array}{lrll} \max & \sum_{e\in E} y_e & & \\ \text{s.t.} & \sum_{e\in\delta(v)} y_e & \leq w_v, & \forall v \in V \\ & y_e & \geq 0, & \forall e \in E \end{array} \tag{M}$$

a) Design a primal-dual algorithm (using the same framework as in the lectures) for solving the minimum weighted vertex cover problem.

b) Prove that it is a 2-approximation algorithm.

c) What is the relation between this result and the result for Set cover?

Exercise 2 Consider the (Minimum) Hitting Set problem:

Definition 2 (Minimum Hitting Set)**.**

Input: *a set U, and a family of subsets $\mathcal{S} = S_1, S_2, \ldots, S_k$ of U.*

Task: *find a hitting set for $\mathcal{S}$, a subset $H \subseteq U$ such that $H \cap S_i \neq \emptyset$ for all $1 \leq i \leq k$.*

Objective: *minimize* $|H|$.

Show that the (unweighted) Set Cover problem discussed in class has a ρ-approximation if and only if the Hitting Set problem has a ρ-approximation.

Exercise 3 (See also Section 7.4 of [WS]) Consider the GENERALIZED STEINER TREE problem:

Definition 3 (Generalized Steiner tree)**.**

Input: *graph* $G = (V, E)$*, distances* $d : E \to \mathbb{R}_+$ *and* k *pairs of nodes* $(s_i, t_i) \in V^2$*.*

Task: *find a subset* $F \subseteq E$*, such that for every* (s_i, t_i)*-pair,* s_i *and* t_i *are connected.*

Objective: *minimize the total length of* F*, i.e. minimize* $\sum_{e \in F} d_e$*.*

a) Give a primal and dual formulation for this problem.

b) Come up with a primal-dual algorithm for the Generalized Steiner tree problem. HINT: this algorithm is very similar to the one for Price-collecting Steiner tree that we saw in the lecture. This includes some cleanup step.

c) Prove that the primal-dual algorithm is a 2-approximation.