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Summer 2017

Approximation Algorithms

Exercise Sheet 5

Exercise 1 (*15 points*) Consider the set cover problem once again. This time, you shall be designing an $\mathcal{O}(\log n)$-approximation algorithm for this problem using primal and dual of an LP relaxation. Given a universe $U = \{e_1, e_2, \cdots e_n\}$ and a family $\mathcal{F} = \{S_1, S_2, \cdots S_m\}$ of subsets of U along with a non-negative cost function $c : \mathcal{F} \to \mathbb{R}^+$. The following is a natural LP relaxation to the problem:

$$\text{minimize} \sum_{S \in \mathcal{F}} x_S \cdot c(S)$$
$$\sum_{S: e \in S} x_S \geq 1, \forall e \in U$$
$$x \geq 0$$

First, construct a dual to the above LP. (*5 points*)

You will find that for each element $e \in U$, there is a dual variable. Now, consider the greedy algorithm for set cover as done in the lectures. In the algorithm, whenever you pick a set, try to assign a dual value to all new elements that the new set covers, such that the total dual you have assigned is able to pay for the cost of the new set. Note that this step is just for doing the analysis and does not affect the algorithm.

a) Is the above constructed dual feasible?

b) If not, try to come up with a suitable scaling factor such that they become feasible. Then use weak duality to show that the total primal cost is at most $\mathcal{O}(\log n)$ times the cost of a feasible dual solution and finish the argument. (*10 points*)

(*Hint:* $1 + 1/2 + 1/3 + \cdots 1/n = \mathcal{O}(\log n)$.)

Exercise 2 (*15 points*) Consider the minimum cost perfect matching problem on a metric graph. Given a complete graph $G(V, E)$, with $|V| = n, |E| = m$, where n is even and a non-negative cost function $c : E \to \mathbb{R}^+$ on edges such that it obeys the metric properties. The task is to find a matching F of minimum cost such that each and every vertex appears as an endpoint of one of the edges in F. The problem has a polynomial time exact algorithm. However, in this exercise, we shall design a 2-approximation to the same using the moat growing algorithm we did for Steiner Forest.

a) (*5 points*) Write a cut-based LP relaxation to the problem. To this end, try to design an LP that has the following constraint: any odd sized cut much have at least one edge going out of it. Argue that finding the minimum cost set of edges that satisfies this is equivalent to finding a perfect matching in the graph of the same or lower cost. (*Hint: Metric property*).

b) (*10 points)* Write a dual to your LP and use the moat growing algorithm to design a 2-approximation to the problem.

Exercise 3 (*10 points*) Design a simple 2-approximation for the bin-packing problem. (*Hint:* Greedy !)