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Approximation Algorithms

Exercise Sheet 1

Exercise 1 Makespan. In the makespan minimization problem, we are given a set of identical machines $M = \{i_1, i_2, \cdots i_m\}$ and a set of jobs $J = \{j_1, j_2, \cdots j_n\}$. The processing size of job j is denotes by p_j. The objective is to determine a schedule that minimizes the maximum load on any machine or, in other words, the makespan.

We saw in the lectures that a greedy algorithm that assigns jobs in arbitrary order to the least loaded machine is a 2-approximation.

a) (*5 points*) Improve the analysis to show that it is indeed a $(2 - \frac{1}{m})$-approximation algorithm, where m is the number of machines.

b) (*5 points*) Prove that the above bound is tight. That is, design a family of instances such that the ratio between makespan produced by the greedy scheduling is $(2 - \frac{1}{m})$ times the makespan of an optimal schedule for that instance.

Exercise 2 (*5 points*) In the lectures we saw that Christofides' Algorithm is a 3/2-approximation for the problem of finding the minimum Travelling Salesman Problem in a complete graph where the cost function obeys the metric properties. Prove that this analysis is essentially tight (for large inputs).

Exercise 3 ***K*-center.** In the K-center problem, we are given a set of n points, $P = \{p_1, p_2, \cdots p_n\}$ in a metric space, i.e., we are given a distance function $d : P \times P \to \mathbb{R}^+$, such that

- $d(p_i, p_j) = d(p_j, p_i)$
- $d(p_i, p_j) \leq d(p_i, p_k) + d(p_k, p_j)$

The goal is to select a subset $C \subseteq P$ of K points such that $\max_i d(p_i, C)$ is minimized, where $d(p_i, C) = \min_{p' \in C} d(p_i, p')$. The problem is NP-Hard (we shall see a brief proof in the Exercise session).

a) (*10 points*) Consider the following algorithm for K-Center. Assume that you have correctly guessed the optimal solution value $D^\star$. Initialize all points as *uncovered*. Now, iteratively select one point as a new center and mark all points at a distance of at most

$2D^\star$ from it, including itself, to be *covered*. Repeat the procedure till all points are marked *covered*.

Observe that the objective value of the above algorithm is at most $2D^\star$. Now prove that we could not have selected more than K centers. This would complete the proof that the above algorithm is a 2-approximation.

b) (*1 point*) Show how to remove the assumption that you know the correct value of $D^\star$?

c) (*4 points*) Find an example to show that the analysis of the above algorithm is tight.